A simplified approach to analyze of active circuits containing operational amplifiers

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A Simplified Approach to Analyze of Active Circuits Containing Operational Amplifiers

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Abstract: In this paper, a systematic and efficient formulation for analysis of active circuits containing operational amplifiers is presented. The modified nodal approach is used in obtaining system equations of active circuits. The model of operational amplifier relating to the used analysis method is given. The model is a matrix-based approach. Therefore, it allows computer-aided analysis of active circuits to be realized efficiently. Application examples are included into the study.

Key words: active circuits, op amp, model, modified nodal analysis

Poenostavljen pristop analize aktivnega vezja z operacijskim ojačevalnikom

Povzetek: V članku je predstavljen sistematična in učinkovita formulacija analize aktivnih vezij z operacijskim ojačevalnikom. Uporabljen je modificiran vozliščni pristop v sistemu enačb aktivnega vezja. Podan je model operacijskega ojačevalnika za uporabljeno metodo analize. Model je na osnovi matrike, kar omogoča učinkovito računalniško podprto analizo aktivnih vezij. Primeri so vključeni v študijo

Ključne besede: aktivna vezja, operacijski ojačevalnik, model, modificirana vozliščna analiza

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1. Introduction

Operational amplifiers (Op amp) are the most important elements of active circuits. They are used in many applications, such as active filters, amplifiers, digital to analog converter and analog to digital converter in circuit analysis and control systems. The ones interested in electrical, electronics and computer engineering generally have difficulties in obtaining system equations of active circuits containing operational amplifiers. It arises from Op amp models used for the formulation.

Singular network elements, nullator and norator, are used for analysis of Op amp circuits in [1]. It is very difficult to understand and realize the analysis with these elements. Wilson proposed a systematic procedure for analysis of Op amp circuits [2]. But, it has some restrictions about dependent sources and some circuit elements. Gottling presented the use of nodal and mesh methods by inspection in the analysis of active circuits [3]. It has a form similar to Wilson's matrix solution. Although it is more general, it involves very intensive mathematical processes and transformations.

In general, it is very suitable to use the modified nodal method for analysis of active circuits. The classical nodal method, before the modified nodal method, is used for both resistive circuit analysis (DC analysis) and dynamic circuit analysis in many introductory electric circuit textbooks [4-7]. The node voltage method using virtual current sources for special cases is realized in [8]. The nodal voltage method is based on a systematic application of Kirchhoff's current law (KCL). In this method, the circuit variables are node voltages. It provides a simple and systematic solution for circuits that contain only independent current sources and resistances/impedances. But, the classical nodal method has some restrictions. Every circuit element cannot be easily included into to the system equations. For analysis with this method, the circuits must not contain dependent sources (excluding voltage-controlled current source) and voltage sources that are not transformable to current sources (independent or dependent). As an extension to the classical nodal voltage method, the modified nodal analysis (MNA) was first introduced by Ho et al [9] to overcome its shortcomings and has been developed more by including many circuit elements (transformer, semiconductor devices, short circuit, etc.) into the system equations so far. In this method, the system equations can be also obtained by inspection. Especially, it is very suitable for computer-aided analysis of active circuits. In this paper, it is shown how to include the terminal equations, the model, of operational amplifier into the MNA system.

The systematic synthesis of operational amplifier circuits is realized by admittance matrix expansion in [10]. Full model and characterization of noise in operational amplifier is given in [11]. Modeling of operational amplifier based on VHDL-AMS is presented in [12]. Several applications, such as filters, amplifiers, relating to Op amps are given in [13-18]. The Op amp is the premier linear active device in present-day analog integrated circuit applications. Therefore, it is very important to model the Op amp for system analysis.

The paper is organized as follows. In Section 2, the modified nodal analysis is explained. Section 3 summarizes the fundamental characteristics of Op amp, before including it into the MNA system. In Section 4, we develop the MNA model of Op amp. Application examples of the approach are given in Section 5. The paper concludes in Section 6.

2. Modified Modal Analysis

The MNA method allows the system equations to be obtained easily and systematically without any limitations. Therefore, it is very understandable analysis method in system analysis. The main advantage is that the system equations can be also obtained by inspection. In this method, there are both voltage variables and current variables. The modified nodal equations in Laplace (s) domain can be written in the following form.

$$[G+sC]X(s)=BU(s) \tag{1}$$

Where, G, C, B are coefficients matrices. All conductance and frequency-independent values arising in the MNA formulation are stored in matrix G, whereas values of capacitors and inductors are stored in matrix C because they are associated with the frequency. Inductors are included in impedance form, capacitors and resistors are included in admittance form into the MNA system. U(s) represents the source vector containing the independent current and voltage sources. X(s) is the unknown vector in s-domain. In this method, in addition to node voltages, currents of inductors, currents of independent and dependent voltage sources are also taken as variables. The idea underlying this formulation is to split the elements into two groups; the first one is formed by elements which have an admittance description and the other by those which do not. Taking into account the types of variables, the unknown vector and coefficient matrices are partitioned as follows.

$$\left\{\begin{bmatrix} G_A & G_{AB} \\ G_{BA} & G_B \end{bmatrix} + s\begin{bmatrix} C_A & 0 \\ 0 & L_A \end{bmatrix}\right\}\begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} = B\begin{bmatrix} E(s) \\ J(s) \end{bmatrix} \tag{2}$$

Where, $X_1(s)$ represents the node voltage variables, $X_2(s)$ represents the current variables. $X_2(s)$ also expresses required additional variables in the formulation of MNA. G_A is conductance matrix. G_{AB} and G_{BA} ($=G_{AB}^T$) are incidence matrices relating to the connection of elements, whose currents are introduced as variables, to the rest of circuit. G_B contains the controlling constants of dependent sources. C_A and L_A are capacitance and inductances matrices, respectively. E(s) and J(s) are independent voltage and current sources. If there are n nodes and m current variables in a circuit, $X_1(s)$ vector contains n–1 nodal voltage variables except reference node (ground) and $X_2(s)$ vector contains m current variables. Thus, the unknown vector X(s) contains n–1+m variables as seen in Eq. (3).

$$X_1(s)=\begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_{n-1} \end{bmatrix}, \quad X_2(s)=\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_m \end{bmatrix} \Rightarrow X(s)=\begin{bmatrix} X_1(s) \\ \ldots\ldots \\ X_2(s) \end{bmatrix}=\begin{bmatrix} U_1 \\ \vdots \\ U_{n-1} \\ \ldots\ldots \\ I_1 \\ \vdots \\ I_m \end{bmatrix} \tag{3}$$

3. Op-Amp Model

Op amp circuits are fundamental building blocks in a wide range of signal processing applications, especially instrumentation, status monitoring, process control, filtering, digital to analog conversion and analog to digital conversion. Before obtaining the MNA model of Op amp, the fundamental properties of Op amp (Fig. 1) should be summarized. An ideal operational amplifier has the following characteristics: infinite gain for differential input signal, zero gain for common mode input signal, infinite input impedance, zero output impedance and infinite bandwidth. The transfer characteristic of Op amp is shown in Fig. 1. b. It explains the relationships between the input voltages (U_p,U_n) and the output voltage (U_o). In the linear region, the input-output relation is

$$U_o = A(U_p - U_n) = AU_d \tag{4}$$

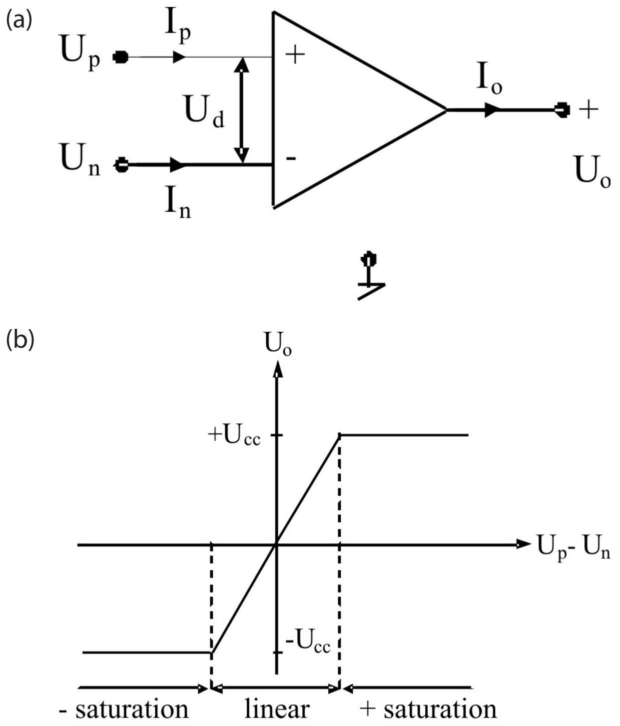

Figure 1: (a) Op amp, (b) Op amp characteristics

In analog integrated circuits, Op amps usually operate in the linear mode. The equivalent circuit model of Op amp operating in its linear range is shown in Fig. 2.a, where R_i is the input resistance, R_o the output resistance. It also contains the voltage controlled voltage source whose gain is A. The ideal Op amp has $R_i=\propto$, $R_o=0$, $A= \propto$ (Fig. 2.b). In the ideal Op amp operating in the linear mode, U_o is limited, the potential difference between input terminals must be zero as A approaches infinity ($A\rightarrow \propto$).

$$U_o = A(U_p - U_n) = AU_d \quad \rightarrow \quad U_d = \frac{U_o}{A} = U_p - U_n = 0 \tag{5.a}$$

$$U_p = U_n \tag{5.b}$$

Since the input resistance of ideal Op amp is infinite, the input currents must be zero.

$$I_n = 0, \quad I_n = 0 \tag{6}$$

According to the Op amp constraints, given in Eq. (5) and Eq. (6), Op amp is a linear and time-invariant device. Because $I_p=I_n=0$ and $U_p=U_n$, the input terminals of Op amp are simultaneously short circuit ($U_p=U_n$) and open circuit ($I_p=I_n=0$). It is an interesting property of the Op amp.

4. MNA model of Op amp

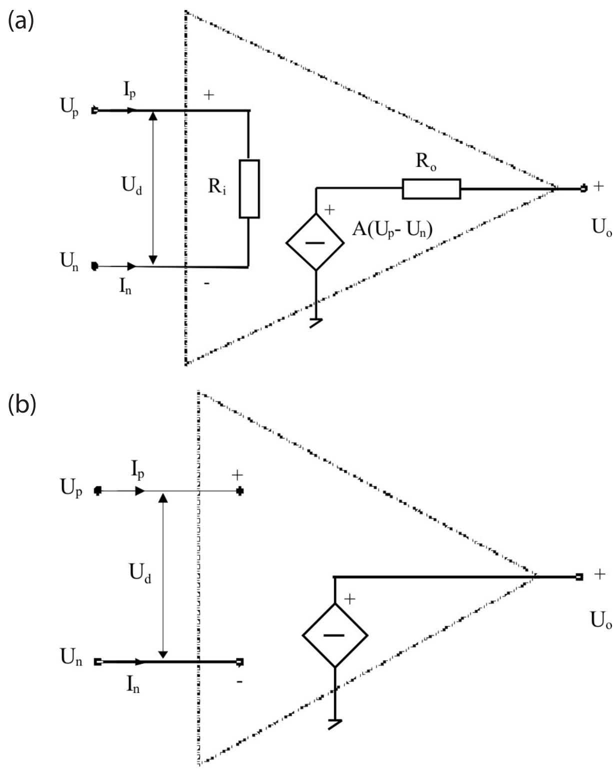

Figure 2: (a) Equivalent circuit of Op Amp, (b) Ideal Op Amp model

The ideal Op amp concept is a good approximation to analyze the Op amp circuits. Therefore, this concept will be used for developing the MNA model of Op amp. For MNA structure, first, the terminal equations of Op amp (Op amp constraints), given in Eq. (5) and Eq. (6), are expressed together as in Eq.(7).

$$\begin{bmatrix} I_p \\ I_n \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} U_p \\ U_n \\ 0 \end{bmatrix} \tag{7}$$

As explained in Section 2, there are m current variables, $X_2(s)$, in the MNA system. I_p, I_n currents of Op amp are located in $X_2(s)$ vector. The short circuit property of input terminals of Op amp is included as an additional equation into the MNA system, as will be explained in the following.

Let an active circuit contain n nodes, including three terminals (nodes) of Op amp. In the MNA system, there are n–1 nodal voltage variables, $X_1(s)$. In the ideal Op amp model (Fig. 2.b), the output voltage of Op amp is determined by other nodal voltages because of the dependent voltage source connected between the output terminal and ground. Therefore, it is not necessary to write a nodal equation at the output node of Op amp. If there is an Op amp in a circuit, we formulate nodal equations at other n–2 nonreference nodes. Since there are n–1 nodal voltages in the system, we seem to have more unknowns than equations. However, the short circuit property of input terminals of Op amp (U_p–U_n=0) supplies an additional equation into the system. If a circuit contains k Op amps, n–1–k nodal equations are formulated, except output terminals of Op amps, and k additional equations relating to the short circuit property of input terminals are included into the system equations.

In Eq. (8), it is shown how to include the terminal equations of Op amp, input currents and short circuit property in Eq. (7), into the MNA system in Eq. (1). The constraints of Op amp are stored in matrices G and B. Eq.(8) gives the MNA model of Op amp. This model contains both the short circuit property (U_p–U_n=0) and the open circuit property (I_p=I_n=0) of input terminals of Op amp.

$$GX(s)+sCX(s)=BU(s)$$

$$\underbrace{\begin{bmatrix} & & & & \cdot & & & & & \\ & & & & \cdot & & & & & \\ & & & & \cdot & & & & & \\ & & & & \cdot & & & & & \\ .. & .. & .. & .. & .. & .. & .. & .. & .. & .. \\ & & & & \cdot & & & & & \\ & & & & \cdot & & & 1 & & \\ & & & & \cdot & & & & 1 & \\ & 1 & -1 & & \cdot & & & & & \\ & & & & \cdot & & & & & \end{bmatrix}}_{G}\underbrace{\begin{bmatrix} \vdots \\ U_p \\ U_n \\ \vdots \\ \cdots \\ \vdots \\ I_p \\ I_n \\ \vdots \\ \vdots \end{bmatrix}}_{X(s)}=\underbrace{\begin{bmatrix} \vdots \\ \vdots \\ \vdots \\ \vdots \\ \cdots \\ \vdots \\ 0 \\ 0 \\ 0 \\ \vdots \end{bmatrix}}_{B}U(s) \quad (8)$$

In the MNA model, given by Eq. (8), the current constraints of the ideal Op Amp concept, I_p=I_n=0, appear to be fairly useless because it draws no currents at its inputs. Therefore, these currents can be ignored when formulating the MNA system in order that the system matrix has min. dimensions, as done in Examples. It is sufficient to take into consideration the short circuit property of input terminals of Op amp. Consequently, the MNA model of Op amp can be also expressed as in Eq. (9). This model can be also included into the system equations by inspection. In the examples of Section 5, for the MNA model of Op amp, Eq. (9) is used in order that the system equations have min. variables.

$$\underbrace{\begin{bmatrix} & & & & \cdot & & & \\ & & & & \cdot & & & \\ & & & & \cdot & & & \\ & & & & \cdot & & & \\ .. & .. & .. & .. & .. & .. & .. & .. \\ & & & & \cdot & & & \\ & 1 & -1 & & \cdot & & & \\ & & & & \cdot & & & \end{bmatrix}}_{G}\underbrace{\begin{bmatrix} \vdots \\ U_p \\ U_n \\ \vdots \\ \cdots \\ \vdots \\ \vdots \\ \vdots \end{bmatrix}}_{X(s)}=\underbrace{\begin{bmatrix} \vdots \\ \vdots \\ \vdots \\ \vdots \\ \cdots \\ \vdots \\ 0 \\ \vdots \end{bmatrix}}_{B}U(s) \quad (9)$$

5. Application Examples

In this section, the analysis of three active circuits containing Op amp are realized by the presented model. The first example is the differential amplifier circuit having two inputs. The second one is the high-pass Butterworth active filter circuit containing energy storage elements (capacitor). The band-pass Butterworth state-variable filter is used to demonstrate the use of Op amps in cascade connection in the last example.

Example 1: Consider the differential amplifier circuit in Fig. 3. It has two input signals.

The circuit has n-1=5 nonreference nodes, including input-output terminals of Op Amp. Thus, in the MNA system, X_1 vector contains 5 nodal voltage variables. Normally, it requires to be obtained an equation for every node. But, it is not necessary to write a nodal equation for output node (node e) because of the features of ideal Op amp, as explained in Section 4. The voltage and current constraints of Op Amp are included into the system equations, as shown in the MNA model of Op Amp in Eq. (8) or Eq. (9). There is no need to put the input terminal currents of the Op amp into the MNA system according to Eq. (9). Therefore, the current variables in X_2 vector are only source currents. They are relating to additional equations.

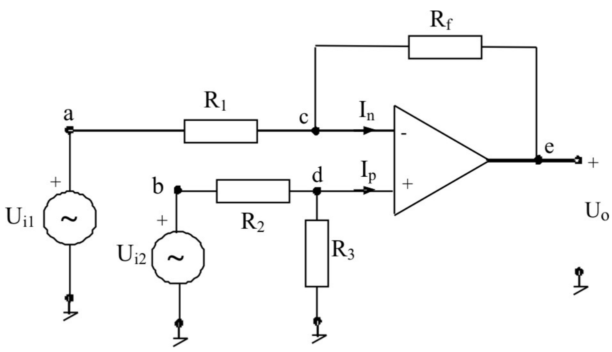

Figure 3: Differential amplifier

The nodal (main) equations of the differential amplifier:

$$a \rightarrow \quad G_1(U_a - U_c) + I_{Ui1} = 0$$

$$b \rightarrow \quad G_2(U_b - U_d) + I_{Ui2} = 0$$

$$c \rightarrow \quad G_f(U_c - U_e) - G_1(U_a - U_c) + I_n = 0$$

$$d \rightarrow \quad G_3 U_d - G_2(U_b - U_d) + I_p = 0$$

Additional equations: $U_c - U_d = 0 \;\rightarrow$ Op Amp constraint, $I_p = 0,\ I_n = 0$

$$U_a = U_{i1}$$

$$U_b = U_{i2}$$

The overall equations constitute the MNA system (Eq. 10). They are represented in matrix form, as in Eq. (1) or Eq. (2). Since the circuit has no storage elements, Matrix C is not available. The MNA model of Op Amp, given by Eq. (9), can be also seen from system equations in Eq. (10).

$$GX(s) = BU(s) \rightarrow \begin{bmatrix} G_A & \vdots & G_{AB} \\ \cdots\cdots & \cdots & \cdots\cdots \\ G_{BA} & \vdots & G_B \end{bmatrix} \begin{bmatrix} X_1(s) \\ \cdots\cdots \\ X_2(s) \end{bmatrix} = BU(s)$$

$$\begin{array}{l} \\ \\ \\ \\ \text{Additional} \\ \text{equations} \end{array} \begin{bmatrix} G_1 & 0 & -G_1 & 0 & 0 & \vdots & 1 & 0 \\ 0 & G_2 & 0 & -G_2 & 0 & \vdots & 0 & 1 \\ -G_1 & 0 & G_1+G_f & 0 & -G_f & \vdots & 0 & 0 \\ 0 & -G_2 & 0 & G_2+G_3 & 0 & \vdots & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & \vdots & 0 & 0 \\ \cdots & \cdots & \cdots\cdots & \cdots\cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & 0 & 0 & \vdots & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & \vdots & 0 & 0 \end{bmatrix} \begin{bmatrix} U_a \\ U_b \\ U_c \\ U_d \\ U_e \\ \cdots \\ I_{Ui1} \\ I_{Ui2} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \cdots & \cdots \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} U_{i1}(s) \\ U_{i2}(s) \end{bmatrix} \tag{10}$$

The output voltage, $U_o = U_e$, is obtained by solving the system equations as follows;

$$U_o(s) = -\frac{R_f}{R_1} U_{i1}(s) + \left(\frac{R_3(R_f + R_1)}{R_1(R_2 + R_3)} \right) U_{i2}(s)$$

Example 2: Consider the high-pass Butterworth filter circuit in Fig. 4. It has two energy storage elements.

The circuit has n–1=5 nonreference nodes, including input-output terminals of Op Amp. Therefore, X_1 vector contains 5 nodal voltage variables. It is not necessary to write a nodal equation for output node (node e) and to put the input terminal currents of the Op amp into the MNA system according to Eq. (9).

The nodal (main) equations of high-pass Butterworth filter circuit**:**

$$a \rightarrow \quad sC_1(U_a - U_b) + I_{Ui} = 0$$

$$b \rightarrow \quad sC_2(U_b - U_c) - sC_1(U_a - U_b) + G_1(U_b - U_e) = 0$$

$$c \rightarrow \quad G_2 U_c - sC_2(U_b - U_c) + I_p = 0$$

$$d \rightarrow \quad G_b U_d - G_a(U_e - U_d) + I_n = 0$$

Additional equations : $U_c - U_d = 0 \;\rightarrow$ Op Amp constraint, $I_p = 0,\ I_n = 0$

$$U_a = U_i$$

The overall equations constitute the MNA system (Eq.11). They are represented in matrix form, as in Eq.(1).

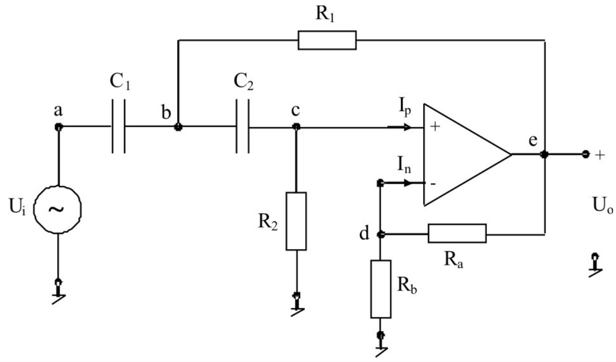

Figure 4: High-pass Butterworth active filter circuit

$$[G + sC]X(s) = BU(s)$$

$$\begin{array}{l} \\ \\ \\ \\ \text{Additional} \\ \text{equations} \end{array} \begin{bmatrix} sC_1 & -sC_1 & 0 & 0 & 0 & \vdots & 1 \\ -sC_1 & G_1+sC_1+sC_2 & -sC_2 & 0 & -G_1 & \vdots & 0 \\ 0 & -sC_2 & G_2+sC_2 & 0 & 0 & \vdots & 0 \\ 0 & 0 & 0 & G_a+G_b & -G_a & \vdots & 0 \\ 0 & 0 & 1 & -1 & 0 & \vdots & 0 \\ \cdots & \cdots\cdots\cdots\cdots & \cdots & \cdots\cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & 0 & 0 & \vdots & 0 \end{bmatrix} \begin{bmatrix} U_a \\ U_b \\ U_c \\ U_d \\ U_e \\ \cdots \\ I_{Ui} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \cdots \\ 1 \end{bmatrix} U_i(s) \tag{11}$$

The output voltage, $U_o = U_e$, is obtained by solving the system equations as follows;

$$U_o(s) = \frac{s^2 R_1 R_2 C_1 C_2 (R_a + R_b)}{s^2 R_1 R_2 R_b C_1 C_2 + s(R_1 R_b C_1 + R_1 R_b C_2 - R_a R_2 C_2) + R_b} U_i(s)$$

Example 3: Consider the band-pass Butterworth state-variable filter circuit in Fig. 5. Here, the use of Op amps in cascade connection is shown. It will be seen how much the model simplify the solution of complex Op amp circuits by dint of its efficient formulation. The state-variable filter uses three Op amps, two integrators and one summing amplifier. The main advantageous of the state variable filter is that it has low-pass (LP), high-pass (HP) and band-pass outputs (BP). In Fig. 5, these outputs are shown as U_{LP}, U_{HP} and U_{BP}, respectively.

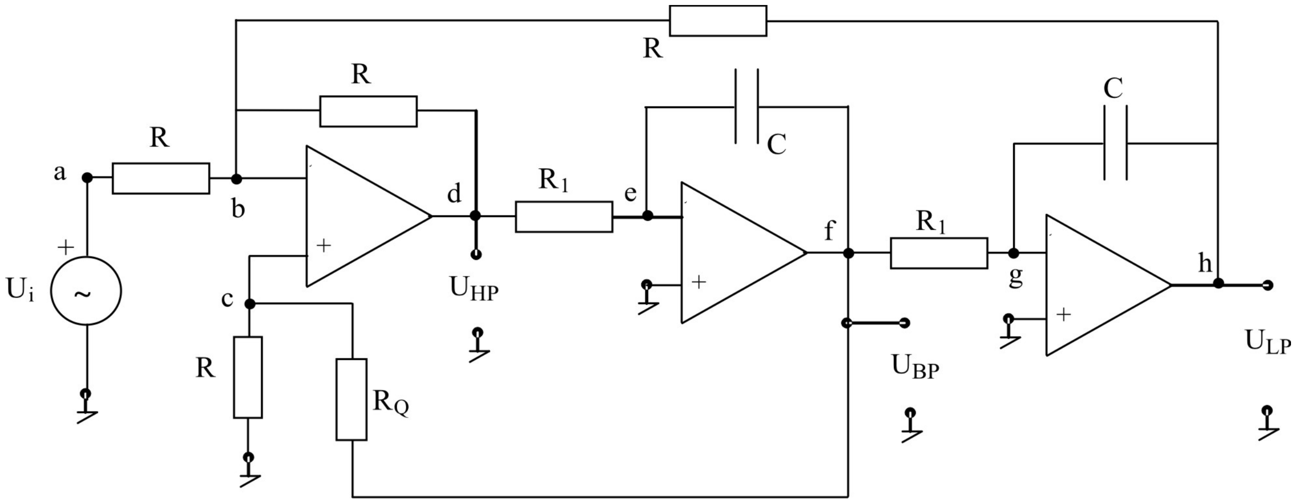

Figure 5: Band-pass Butterworth state-variable filter circuit

The circuit has n-1=8 nonreference nodes, including input-output terminals of Op Amps. Therefore, X_1 vector contains 8 nodal voltage variables. It is not necessary to write nodal equations for output nodes (node d, f, h) and to put the input terminal currents of the Op amps into the MNA system according to Eq. (9). In the system equations and Fig. 5, these currents are not shown.

The nodal (main) equations of the circuit**:**

$$a \rightarrow \quad G(U_a - U_b) + I_{Ui} = 0$$

$$b \rightarrow \quad G(U_b - U_d) - G(U_a - U_b) + G(U_b - U_h) = 0$$

$$c \rightarrow \quad GU_c + G_Q(U_c - U_f) = 0$$

$$e \rightarrow \quad G_1(U_e - U_d) + sC(U_e - U_f) = 0$$

$$g \rightarrow \quad G_1(U_g - U_f) + sC(U_g - U_h) = 0$$

Additional equations: $U_b - U_c = 0 \quad , U_e = 0, \quad U_g = 0$

$$U_a = U_i$$

The overall equations constitute the MNA system (Eq.12).

$$\begin{array}{c} \\ \\ \\ \\ \\ \text{Additional equations} \left\{ \begin{array}{c} \\ \\ \\ \\ \\ \end{array} \right. \end{array} \begin{bmatrix} G & -G & 0 & 0 & 0 & 0 & 0 & 0 & \vdots & 1 \\ -G & 3G & 0 & -G & 0 & 0 & 0 & -G & \vdots & 0 \\ 0 & 0 & G+G_Q & 0 & 0 & -G_Q & 0 & 0 & \vdots & 0 \\ 0 & 0 & 0 & -G_1 & G_1+sC & -sC & 0 & 0 & \vdots & 0 \\ 0 & 0 & 0 & 0 & 0 & -G_1 & G_1+sC & -sC & \vdots & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & \vdots & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \vdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \vdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \vdots & 0 \end{bmatrix} \begin{bmatrix} U_a \\ U_b \\ U_c \\ U_d \\ U_e \\ U_f \\ U_g \\ U_h \\ \cdots \\ I_{Ui} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \cdots \\ 1 \end{bmatrix} U_i(s) \qquad (12)$$

The filter outputs, $U_{LP}=U_l$, $U_{HP}=U_d$, $U_{BP}=U_g$ are obtained by solving the system equations as below.

$$U_{LP}(s) = U_h(s) = \frac{-(R_Q + R)}{s^2 R_1{}^2 C^2 (R_Q + R) + 3sRR_1C + R_Q + R} U_i(s)$$

$$U_{HP}(s) = U_d(s) = \frac{-s^2 R_1{}^2 C^2 (R_Q + R)}{s^2 R_1{}^2 C^2 (R_Q + R) + 3sRR_1C + R_Q + R} U_i(s)$$

$$U_{BP}(s) = U_f(s) = \frac{sR_1C(R_Q + R)}{s^2 R_1{}^2 C^2 (R_Q + R) + 3sRR_1C + R_Q + R} U_i(s)$$

6. Conclusion

The main difficulty in obtaining the system equations of active circuits containing Op Amps in system analysis arises from Op Amp models used for the formulation. In this paper, an efficient and systematic approach for analysis of active circuits containing Op Amps has been presented. The modified nodal approach, very understandable analysis method, is used in obtaining the system equations. The fundamental characteristics of Op amp have been summarized and the MNA model of Op amp has been developed. As a result, a matrix-based framework for computer-aided analysis of active circuits has been formulated. The model is general, systematic and can be applied to all possible active circuit structures. Examples are included to show the efficiency of the analysis method and the MNA model of Op amp. Using the presented model, it can be easily obtained system equations of Op Amp circuits by inspection and also, can be written a computer program about analysis of active circuits.

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